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APPLICATION OF A SIMPLE SPACE-TIME AVERAGED POROUS MEDIA MODEL FOR FLOW IN DENSELY VEGETATED CHANNELS

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ABSTRACT

Traditional flow modelling in open channels uses time averaged turbulence models. These models are valid in clear fluid, but not if dense obstructions are present in the flow field. In this presentation we show that newly developed models can describe the fluid flow like flow in a porous medium. Clear fluid models don't take into account drag due to the presence of the obstacles. Flow in rivers, channels, estuaries and irrigation networks is often obstructed by vegetation, and coarse bedrock. In computer modelling applications appropriate turbulence resistance models are either absent or empirically based. In this paper we develop a space–time averaged form of the Navier–Stokes equations, in order to improve modelling of flow in densely obstructed channels. We use a combination of Reynolds averaging for the turbulent flow and volume averaging in order to take into account the dense obstructions. We show that the obstacle density can be modelled by a porosity term if structural parameters of the vegetation are taken into account. In order to take the structural parameters of the vegetation into account we develop a Representative Unit Cell (RUC) concept, borrowed from volume averaging in porous media. Inside the RUC local flow solutions for the Navier–Stokes equations are developed and used as closure terms in the space–time averaged form of the Navier–Stokes equations. Our expression depends on measurable quantities like average porosity and average vegetation diameter. It can be used in computational fluid dynamics models to directly include vegetation characteristics instead of approximate resistance factors. As an application we use our theoretically derived model to compute resistance factors for Manning's equation from the structural properties of the vegetation modeled as a porous medium.

NOMENCLATURE

A Surface of fluid interface with solid $[m^2]$.
A_f Streamwise area availiable for fluid flow $[m^2]$.
A_p Frontal area of plant stems $[m^2]$.
C_D Drag coefficient of individual plant stems $[-]$.
d RUC length $[m]$.
d_s Stem diameter of waterplant $[m]$.
D Characteristic length $[m]$.

F_D Drag force on plant stem $[kg/ms^2]$.
g Gravity acceleration $[m/s^2]$.
n Constant in Mannings equation $[s/m^{1/3}]$.
p Pressure $[kg/ms^2]$.
R Hydraulic radius $[m]$.
Re Reynolds number $[-]$.
S friction slope $[-]$.
v Velocity $[m/s]$.
V_0 Volume of averaging volume $[m^3]$, d^3.
ε Porosity $[m^3/m^3]$.
ρ Density $[kg/m^3]$.
μ Dynamic viscosity $[kg/ms]$.
subscripts:
m microscopic
symbols:
$\langle(\ )\rangle$ Average of quantity over Volume of RUC

INTRODUCTION

The flow of water in watercourses is a dynamic equilibrium between momentum generation by gravity and dissipation through friction. This friction is mainly influenced by the geometry of watercourse. Rough boundaries increase this friction and so influence the flow resistance. Growth of aquatic plants causes major changes in the flow resistance in water courses. This can have large impact on water levels and flooded areas. In the last years more and more wetland restoration and management schemes are put into use. These are intended to provide a more nature related management of water resources. Water authorities are changing their policies with regard to mowing and removal of aquatic plants. Still the growth of aquatic weeds can cause severe problems in the management and maintenance of irrigation and drainage channels. Another application area of studies of flow resistance of aquatic plants are changing river flow regimes due to urbanisation or possible climate change. These require new assessment of peak flow resistance of vegetated river banks. Aquatic plants can also act as sediment filters and as such can remove sediments from the flowing water [4]. This can substantially reduce the cost in maintaining sand traps and inverted siphons in irrigation channels. Mostly methods based on experience and empirical research based on Mannings equation are used to predict the flow resistance in open channels.
Since the early 1970's research on the flow resistance of aquatic plants developed new methods to described this flow resistance. These methods were based on modifications of the Manning equation and contained various empirical parameters [17]. Thereafter methods based on dimensional analysis and mechanistic reasoning were developed [23]. Recent developments in the volume averaging theory for flow in porous media allow the explicit description of turbulent flow phenomena [12; 15]. In this article we describe one possibility to use porous media based models to quantify resistance coefficients. Our model is restricted partially submerged plants with a stem geometry like reed (*phragmites*) and high porosities.

FLOW MODELS FOR VEGETATION RESISTANCE

Commonly the resistance to water flow in open channels can be split up in different parts: the resistance caused by the rough channel bottom and channel sides and the resistance caused by the water plants [23]. Usually the resistance due to the plants is about an order of magnitude larger than the bottom resistance. In this article we are only concerned with the resistance caused by the water plants.
Our aim is to develop a model of flow resistance for unsubmerged or partially submerged plants. The flow in channels with submerged plants can be described by boundary layer type equations [20]. There exist practical engineering equations to predict the flow resistance under such circumstances [8]. This is in contrast to unsubmerged plants.
If the vegetation is dense, the water flow can be reduced by up to 97% through the additional vegetation resistance. Generally this resistance due to vegetation can be described in terms of viscous drag and pressure drag [23]. Viscous drag plays an important role at low velocities, whereas at higher velocities pressure drag is dominant. Dense vegetation

is like a porous medium, and the influence of the channel bottom will be very small. Inside this porous medium there is no significant boundary layer development due to the bottom influence [10].
The flow can be characterised by the Reynolds number:

$$Re = \frac{v\rho D}{\mu} \tag{1}$$

with v time and space averaged velocity, D a characteristic length scale, ρ the density of the fluid, and μ the kinematic viscosity. For open channel flow D is usually chosen as the height of the water level. This choice of length scale characterises the flow as fully turbulent. This is valid if the channel is not densely obstructed. If a large concentration of water plants is present, it may be more appropriate to use a D in the order of the stem diameter of the water plant as a characteristic length. For typical reed plants and water velocities the Reynolds number is then in the range of $10^3 - 10^4$.
In engineering practice most flow models are of the Manning equation type:

$$v = \frac{1}{n} R^{2/3} S^{1/2} \tag{2}$$

with n Mannings resistance coefficient, R the hydraulic radius, and S the friction slope. Many vegetation resistance models are based on modification of n in Mannings equation. The estimation of n is done through tables [18] or via experiments as is in [23]. Several authors tried to estimate Mannings n through a force balance approach between gravitational force and drag force on the vegetation. Petryk [17] and Kadlec [7] used a C_d approach based on the drag of individual plants terms.
Tsihrintzis [21] correlated available literature data to the total drag coefficient for vegetation. A different approach was used by Kouwen [9]. After dimensional analysis they obtained a relation for the Darcy-Weisbach friction factor in relation to vegetation characteristics. Another approach consists of modelling the vegetation resistance by drag models [7; 17; 9]. These models are often based on dimensional analysis.
More advanced models try to resolve the flow structure inside the open channel and are based on turbulent flow modelling for clear fluids [10]. Much of this work can be traced back to work done in boundary layer meteorology on air resistance of plants [5; 14]. Detailed modelling of turbulent flow over rough boundaries [13] suggested that a combination of spatially or volume averaging and time averaging should be used for modelling and the determination of resistance factors. The closure use of these models is still based on empirical correlations for the drag coefficient of the vegetation.
Still the above models are either based on measurements (the Manning models) or very complicated to use in practice (the turbulent flow models). The drag models are somewhat intermediate in difficulty of use, but lack of reliable and general drag coefficients that make them unsuitable for practical engineering use. In order to mitigate these problems, we average the Navier Stokes equation in space and time. After this we incorporate the vegetation structure in our flow model and simplify it, so that it can be used in engineering practice.

AVERAGING OF THE NAVIER STOKES EQUATIONS

The Navier Stokes equations as such can be used to describe flow through aquatic plants. If this is done, the individual geometry of each plant needs to be resolved. This results in a computational problem of major complexity. From a practical point of view, it is impossible to survey a vegetated channel exhaustively. Average vegetation parameters can be measured or estimated. These parameters are amongst others: vegetation density, average stem diameter, average vegetation geometry, and vegetation height. In order to use these parameters in the Navier Stokes equations the spatial scale of the equation needs to match the spatial scale of the observations.
We use volume averaging [22], which is commonly used in porous media flow. The volume averaged equations are also averaged in time, analogous to the standard Reynolds averaging procedure. Different authors in the porous media literature averaged the Navier Stokes equations for high Reynolds number flow and large porosities. One of the simplest volume averaged models to date is the model by Masuoka [11]. They use a 0-equation model, parameterised

by the sum of the eddy diffusivities caused by two types of vortices (i) the pseudo vortex of the order of the porous medium particle diameter, and (ii) the interstitial vortex between the solid particles. In their model no explicit time averaging is done. Nakayama [12] developed a complete two equation model for macroscopic flow in a porous medium. They start with a Reynolds averaged micro model and after this, they apply the volume averaging procedure. Antohe and Lage [1] use a standard eddy diffusivity concept. Their main result is that at low porosities the porous medium effectively damps the turbulence strongly, but that at higher porosities the porous medium even can enhance the turbulence levels. In addition, their model demonstrates that the only possible solution for steady unidirectional flow is zero macroscopic turbulence kinetic energy. Recently Pedras and Lemos [15] showed that the sequence of averaging operations on the Navier Stokes equations is immaterial. They showed this in a series of articles [16; 15]. In the following we use their approach for the averaging procedure.

For the description of turbulent flow in a porous medium we start with the microscopic mass balance equation and Navier Stokes equation. They are valid on the microscale, i.e. the space in between individual water plants. Details of the averaging procedure can be found in [15]. Assuming incompressible flow, the mass balance equation from the micro to the macro scale becomes:

$$\nabla \cdot v_m = 0 \tag{3}$$

$$\nabla \cdot (\varepsilon v) = 0 \tag{4}$$

with ε the porosity, i.e. the fraction of volume occupied by the fluid.

The microscopic Navier Stokes equations are:

$$\rho\left(\frac{\partial v_m}{\partial t} + \nabla \cdot (v_m v_m)\right) = -\nabla p_m + \mu \nabla^2 v_m + \rho \mathbf{g} \tag{5}$$

And after time and volume averaging they become [15]:

$$\rho \left[\frac{\partial}{\partial t}(\varepsilon v) + \nabla \cdot (\varepsilon \langle \overline{vv} \rangle^i)\right] = -\nabla(\varepsilon p) + \mu \nabla^2 (\varepsilon v) + \nabla \cdot (-\rho \varepsilon \langle \overline{v'v'} \rangle^i) + \varepsilon \rho \mathbf{g} + \mathbf{R} \tag{6}$$

with the terms in angular brackets denoting averaged velocity correlation terms, and **R** given by:

$$\mathbf{R} = \frac{\mu}{V_0} \int_A n \cdot (\nabla v_m)\, dA - \frac{1}{V_0} \int_A n p_m\, dA \tag{7}$$

with n the outward normal unit vector on the surface A. The **R** term (eq. 7) describes the interaction of the flow with the porous medium. In the next section this term is modelled in terms of physical parameters of the porous medium.

SIMPLIFIED GEOMETRICAL MODEL AND CLOSURE

In order to approximate the integrals in (eq. 7), which still contains microscopic values, we use the Representative Unit Cell (RUC) concept of du Plessis [2]. The RUC is a typical geometrical configuration, which captures the relevant flow phenomena inside the porous medium. For our application we choose to model water plants by circular cylinders. With this choice we restrict ourselves to partially submerged reed like plants (*phragmites*). The choice of our closure model was facilitated by the typical very high porosities of aquatic plants in channels. This implies that the individual C_D values of the plant stems can be directly employed in the closure. These C_D values are not identical to C_d values of a single stem in a free stream, but influenced by upstream stems [19].

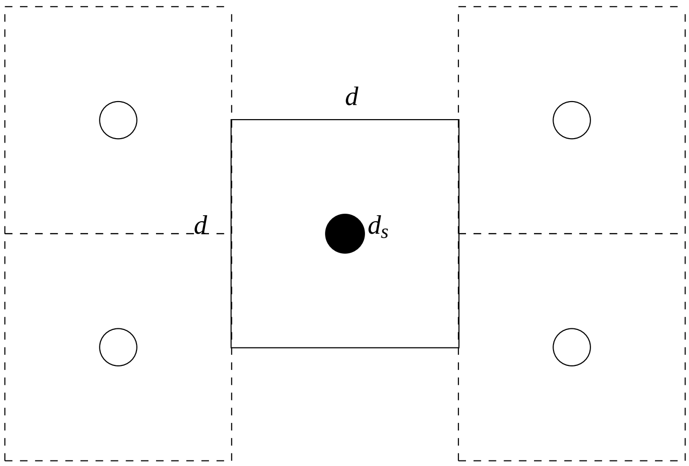

Figure 1. Representative Unit Cell (with neighbouring cells) and dots denoting reed stems

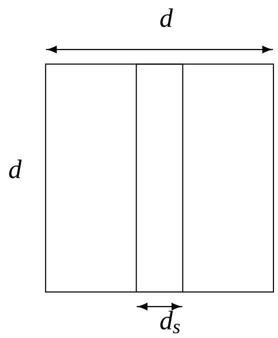

Figure 2. Representative Unit Cell, frontal view

Geometry of the RUC

To keep our model simple, the plant stems are modelled as vertical cylinders with diameter d_s. In figure 1 a 2-D projection of a typical RUC is drawn together with neighbouring cells. Figure 2 shows a frontal view of a RUC. The definition of the porosity ε and other geometric factors is based on:

$$V_0 = d^3 \tag{8}$$

$$V_{solid} = \frac{\pi}{4} d_s^2 d \tag{9}$$

$$V_{fluid} = d^3 - \frac{\pi}{4} d_s^2 d \tag{10}$$

$$A_p = d_s d \tag{11}$$

$$A_f = d(d - d_S) \tag{12}$$

and following:

$$\varepsilon = \frac{V_{fluid}}{V_0} = 1 - \frac{\pi d_S^2}{4d^2} \tag{13}$$

and with the linear dimension d expressed in stem radius and porosity:

$$d = \sqrt{\frac{\pi}{4}} \frac{d_s}{\sqrt{1-\varepsilon}} \tag{14}$$

These geometrical terms are used to characterise the vegetation.

Velocity Relations

The velocity v in equation (6) refers to the space and time averaged velocity inside the RUC, whereas in equation (7) still the microscopic velocity is present. This microscopic velocity v_m differs from v in magnitude and direction, v_m flows around the solid phase, whereas v does not ”flow” around these, due to its averaged nature. This difference in magnitude is caused by the tortuous flow path and the reduced volume available for the flow. Following [3], they can be expressed in each other by:

$$v_m = \frac{v}{\varepsilon} \frac{V_f}{A_f d} = v \left(1 - \sqrt{(1-\varepsilon)(\frac{4}{\pi})} \right)^{-1} \tag{15}$$

with A_f the effective streamer area available for the fluid flow.

Closure

Closure refers to replacing the microscale variables in the integrals in eq. (7) by averaged macroscale quantities. Equation (7) represents the resistance to flow per unit volume due to the vegetation. This resistance depends on the local flow around the plant stems. We choose to express the combined influence of viscous drag (first part of eq. (7)) and pressure drag (second part of eq. (7)) in a combined drag force approach:

$$\mathbf{R} = -\frac{F_D}{V_0} \tag{16}$$

We now seek an expression for F_D. Using the definition of $C_D = \frac{F_D}{0.5\rho v^2 A_p}$ [19], equation (16) becomes:

$$\mathbf{R} = -\frac{1}{V_0} \frac{1}{2} \rho C_D v_m^2 A_p \tag{17}$$

Using the geometrical relations in equations (8), (13), (14), and some manipulation:

$$\mathbf{R} = -\frac{\rho C_D (1-\varepsilon) v^2}{2 d_s (\sqrt{\pi/4} - \sqrt{1-\varepsilon})^2} \tag{18}$$

It remains to specify C_D, the drag coefficient for the individual plant stems. We use the correlation by Taylor [19], who used a discrete element approach to derive following:

$$\log C_D = -0.125 \log Re + 0.375 \tag{19}$$

for $Re < 6 \times 10^4$, and $C_D = 0.6$ for $Re \geq 6 \times 10^4$, Re based on the average velocity.

FINAL EQUATION AND COMPARISON WITH OTHER MODELS

Combining equation (6) and equation (17) we get the following space and time averaged equation:

$$\rho\left[\frac{\partial}{\partial t}(\varepsilon v)+\nabla\cdot(\varepsilon\langle\overline{vv}\rangle^i)\right]=-\nabla(\varepsilon p)+\mu\nabla^2(\varepsilon v)+\nabla\cdot(-\rho\varepsilon\langle\overline{v'v'}\rangle^i)+\varepsilon\rho\mathbf{g}-\frac{\rho C_D(1-\varepsilon)v^2}{2d_s(\sqrt{\pi/4}-\sqrt{1-\varepsilon})^2} \tag{20}$$

The main obstacle in using equation (20) is the inertia term containing fluctuations. It needs to be modelled with an appropriate turbulence model inside the vegetation. The presence of the vegetation restricts the length scale of the eddies that develop and influences the flow through vortex shedding.

For the purpose of engineering applications, and compare it to measurements from literature, we simplify equation (20). By assuming steady, stationary flow, hydrostatic pressure distribution, and flow is driven by the bed slope gradient (then **g** can be replaced by gS):

$$\varepsilon g S=\frac{C_D(1-\varepsilon)v^2}{2d_s(\sqrt{\pi/4}-\sqrt{1-\varepsilon})^2} \tag{21}$$

Equation (21) can be written in a form similar to Mannings equation:

$$v=\sqrt{\frac{2d_s\varepsilon g\left(\sqrt{\pi/4}-\sqrt{1-\varepsilon}\right)^2}{C_D(1-\varepsilon)R^{4/3}}}R^{2/3}S^{1/2} \tag{22}$$

R is the hydraulic radius of the channel and is commonly taken as the depth of flow [6; 23]. The expression for n follows then:

$$n=\sqrt{\frac{C_D(1-\varepsilon)}{2d_s\varepsilon g\left(\sqrt{\pi/4}-\sqrt{1-\varepsilon}\right)^2}}\,R^{2/3} \tag{23}$$

This expression for n can be compared to the work of [6; 23]. Their expression for n rewritten in the current notation and cylindrical plant stems is:

$$n=\sqrt{\frac{C_d(1-\varepsilon)}{2d_s g}}\,R^{2/3} \tag{24}$$

Both expressions for n are dependent on $R^{2/3}$ and depend directly on the vegetation density through ε and the stem diameter. The definitions of C_D and C_d differ. The C_d of [6; 23] must be measured, whereas our C_D is given by equation [19]. In the denominator we have the additional factor $\varepsilon\left(\sqrt{\pi/4}-\sqrt{1-\varepsilon}\right)^2$. This factor is due to the correction for the tortuous flow path through the RUC and the fraction of volume occupied by the plants.

CONCLUSIONS

The use of volume averaging, commonly applied in porous media modelling, can also be applied to open channel flow. In this article preliminary results are given for the resistance coefficient for Mannings equation in densely vegetated channels. The results of the volume averaging technique are similar to the results derived by traditional methods [6; 23]. We show that volume averaging introduces factors for the correction of the tortuous flow path and the fraction of volume occupied by the water plants. These factors are not explicitly taken care of in the traditional equations. This is These first results are promising, and we are working to extend and verify our model.

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